Progressing Along a "Road Less Traveled": The History of School Mathematics

M. A. (Ken) Clements
Illinois State University
<clements@ilstu.edu.>

I shall be telling this with a sigh
Somewhere ages and ages hence:
Two roads diverged in a wood, and I—
I took the one less traveled by,
And that has made all the difference.

Robert Frost, 1916

Much of my own University of Melbourne PhD thesis (Clements, 1979) was concerned with the history of school mathematics in Victoria. Since completing that work, I have gradually extended my knowledge of the history of school mathematics so that it now encompasses much wider international perspectives than ever before. I took "the road less traveled."

During my eight-year period at Monash University (1974-1982) I taught a graduate course on the history of school mathematics, and for over 40 years now, I have taken every opportunity to extend my knowledge on the history of mathematics education in all Australian states, in Europe, in Asia, and finally in North America.

In 1988, Nerida Ellerton and I had an article published on the history of school mathematics in Australia in a special issue of the *Australian Journal of Education* which commemorated 200 years of European settlement in Australia. A few years later, Deakin University published my history of school mathematics in Victoria (Clements, 1991). Then, during a seven-year stint in Brunei Darussalam, I researched the history of school mathematics in Southeast Asia (see, e.g., Horwood & Clements, 2000). Over the past five years, Nerida Ellerton and I have had two books published on the history of school mathematics in North America (Clements & Ellerton, 2015; Ellerton & Clements, 2012; 2014), and another on the history of school mathematics in England (Ellerton & Clements, 2017). Another Springer book, which deals with the history of school algebra from an international perspective (Kanbir, Clements, & Ellerton, in press), is about to appear. I think that this last-mentioned book opens up a whole new area of academic interest – specifically, the history of school mathematics curricula, written from international perspectives.

The first chapter (Clements, Keitel, Bishop, Kilpatrick, & Leung, 2013) of Springer's *Third International Handbook of Mathematics Education*, for which I had overriding editorial responsibility, provided an overview of the history of school mathematics. It was not an easy chapter to write, for nothing similar was available for reference. Each of the four sections in that *Third International Handbook* was structured so that the first chapter was specifically concerned with the history of the main theme addressed by that section.

The way I see things, my journey down the "road less traveled" was justified when in 2016, Nerida and I were honored to be appointed editors of Springer's special series on the history of mathematics education. I am very pleased that one of the early books in that series was written by Australian and Papua New Guinea authors (Owens, Lean, Paraide, & Muke, 2017). I think it would be fair to say that my major legacy to mathematics education

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 43–46). Melbourne: MERGA.

has been, and will continue to be, in the field of the history of mathematics education, and especially the history of school mathematics.

However, I am fully aware that my "road less traveled" is still one which has relatively few travelers. Towards the end of this paper, I argue that it has been my own experience that in mathematics education research, providing a strong historical base for a study is as important as providing a strong "theoretical framework".

The Need for Well-Researched Historical Bases for Mathematics Education Research

There is a tradition in mathematics education of requiring graduate students who are preparing a research thesis to provide a clear theoretical framework for their study. However, there is no firmly established tradition requiring researchers to provide a historical framework for their proposed research, and I am persuaded that there should be.

As mentioned above, in Springer's *Third International Handbook of Mathematics Education* (Clements, Bishop, Keitel, Kilpatrick, & Leung, 2013), there were four major sections, each dealing with a different theme, and each section was structured on the basis of past, present and future aspects of the theme. The first chapter in each section was concerned with analyses of antecedents ("How did we get to where we are now?"); the "middle" chapters provided analyses of present-day key issues for the theme ("Where are we now, and what recent events have been especially significant?"); and the final chapter in each section reflected on future policy ("What should we do to improve the quality of the teaching and learning in the future?"). One of the reasons the editorial team for the *Third International Handbook* adopted this past-present-future organizational structure was to suggest the kind of structure that ought normally to be present in high-quality mathematics education research.

In one of the papers at this 40th annual MERGA conference, Nerida Ellerton, Sinan Kanbir, and I offer a historical perspective on the purposes of school algebra. That paper is based on Chapter 2 of a forthcoming book (Kanbir, Clements, & Ellerton, in press). That chapter provides a substantial overview of the history of school algebra viewed from international vantage points. The last two chapters of the Kanbir et al. (in press) book look to the future, and in the intervening chapters we describe an intervention study in which Sinan Kanbir set out to improve present practices in relation to middle-school algebra. Hopefully, this past-present-future way of thinking about mathematics education research will become more commonly accepted and practised. Certainly, in the United States of America, we have not found many mathematics education studies in which more than peripheral reviews of historical antecedents are provided.

The lack of high-quality histories of school mathematics written from fully internationalized perspectives is a serious matter given that the seventeenth, eighteenth, and nineteenth centuries were marked by massive colonisation programs, whereby the colonisers (mainly European nations) tended to introduce school mathematics textbooks into their colonies, and the languages used in most of those textbooks were those of the colonising powers. The chief authors of the textbooks were, almost always, based in Europe, and textbooks were written which seemed to suggest that school mathematics should be a culture-free exercise. Even for students in the European homelands, the textbooks were designed to suit the perceived needs of children of elites. The first school mathematics textbooks used in the "colonies" were usually written from high mathematics vantage points and, I would argue, were entirely unsuited to meet the needs of indigenous children, or of children of convicts, or of the children of slaves, or of other children whose

command of the spoken and written language used by mathematics teachers or authors of mathematics textbooks was not strong (Clements, Grimison, & Ellerton, 1989).

There were attempts to change the situation. In 1855, in Victoria, Australia, for example, two professors (Professor Hearn and Professor Wilson) at the recently-established University of Melbourne, wanted to create university-entrance regulations which were different from those of the old "home" universities in Europe. The professors argued:

> There are many parents who wish their sons to enter life at an early age but would gladly send them to the University if they could obtain there the amount and quality of education which they wish them to acquire. Such persons think that the study of the classics or the higher mathematics is a needless expenditure of time, and that these subjects, while they have no direct bearing upon their children's future occupations, tend to distract young men from, and give them a distaste for, more practical pursuits. The soundness of such views is not the question. If such an opinion exists, and it is prevalent at home, and probably still more so here, the making these studies a *sine qua non* for a degree would amount to a practical exclusion of the class to whom I have referred. (University of Melbourne, 1855)

However, the colonial conservatives who would administer the yet-to-be-opened University of Melbourne did not approve of such a radical point of view, and they decided that passes in Greek, Latin, Arithmetic, Algebra, and Euclid, at the university's matriculation examination, would be required of all persons wishing to take degrees. Although the course prescribed for matriculation, Algebra, for example, only went as far as "quadratic equations in one unknown" (Clements et al., 1989), any idea that algebra should be "for all children" was not part of the thinking of those who administered the university.

The above University of Melbourne episode draws attention to the need to recognize that, from its beginnings, school mathematics in Australia was, *by design*, not intended for everyone. Secondary school mathematics, in particular, was for "the chosen" (Sharp, Farr, Farr, & Farr, 1936). That was the intention, and any respectable history of school mathematics should make that clear. It took centuries before the idea that school mathematics might be for all would be put forward with any degree of conviction. And, even when that did occur, the challenge of unravelling the forms of mathematics education which, over the centuries, had taken root as "normal", was something which society had to face – usually against staunch opposition from those who wanted to maintain the status quo.

The subconsciously-held traditions of what was normal led to the prescription of forms of school mathematics which were not suited to the needs and backgrounds of many of the students who would be required to study it. In order to study the history of school mathematics adequately, one needs not only to take account of the *intended* curriculum (as summarized in textbooks, and in formal curriculum statements prepared by local, state, or national education authorities), but also the *implemented* curriculum (as represented in cyphering books, or workbooks, or what transpired in mathematics classes), and the *received* curriculum (as represented by student recollections, and data from tests and examinations) (Westbury, 1980). There is a need to regard bottom-up, school-based, perspectives on the history of school mathematics as being as important as top-down, purely mathematics-based perspectives.

I close by asking you to reflect on how you react to the following statement that Nerida Ellerton and I recently felt moved to make:

> We confess to feeling frustrated when, from time to time, we mix with mathematicians who want to colonise the history of school mathematics so that it becomes little more than a study of contributions which great mathematicians have made to mathematics education. We feel equally

> frustrated with mathematics teachers and educators who think that the study of the history of school mathematics beyond, say, the "New Math(s)," or the "national curriculum," or the "NCTM *Standards*," is an extravagant use of time when so much needs to be done to help to improve the existing state of school mathematics. (Ellerton & Clements, 2014, p. 337)

"The history of mathematics education" should be one of MERGA's research themes.

References

Clements, M. A. (1979). *Relationships between the University of Melbourne and the secondary schools of Victoria, 1890–1912* (Unpublished doctoral dissertation). Melbourne, The University of Melbourne.

Clements, M. A. (1992). *Mathematics for the minority: Some historical perspectives on school mathematics in Victoria.* Geelong, VIC: Deakin University.

Clements, M. A., Bishop, A. J., Keitel, C., Kilpatrick, J., & Leung, F. (Eds.). (2013). *Third international handbook of mathematics education*. New York, NY: Springer.

Clements, M. A., & Ellerton, N. F. (2015). *Thomas Jefferson and his decimals 1775-1810: Neglected years in the history of U.S. school mathematics.* New York, NY: Springer.

Clements, M. A., Grimison, L, & Ellerton, N. F. (1989). Colonialism and school mathematics in Australia, 1788-1988. In N. F. Ellerton & M. A. Clements (Eds.), *School mathematics: The challenge to change* (pp. 50-78). Geelong, VIC: Deakin University.

Clements, M. A., Keitel, C., Bishop, A. J., Kilpatrick, J., & Leung, F. (2013). From the few to the many: Historical perspectives on who should learn mathematics. In M. A. Clements, A. J. Bishop, C. Keitel, J. Kilpatrick, & F. Leung (Eds.), *Third international handbook of mathematics education* (pp. 7-40). New York, NY: Springer.

Ellerton, N. F., & Clements, M. A. (1988). Reshaping school mathematics in Australia 1788-1988. *Australian Journal of Education, 32*(3), 387-405.

Ellerton, N. F., & Clements, M.A. (2012). *Rewriting the history of school mathematics in North America 1607-1861: The central role of cyphering books.* New York, NY: Springer.

Ellerton, N. F., & Clements, M. A. (2014). *Abraham Lincoln's cyphering book and ten other extraordinary cyphering books*. New York, NY: Springer.

Ellerton, N. F., & Clements, M. A. (2017). *Samuel Pepys, Isaac Newton, James Hodgson and the beginnings of secondary school mathematics: A history of the Royal Mathematical School at Christ's Hospital 1673-1868*. New York, NY: Springer.

Horwood, J., & Clements, M. A. (2000). A mirror to the past: Mathematics education in Australia and Southeast Asia: Past, present and future. In M. A. Clements, H. Tairab, & K. Y. Wong (Eds.), *Science, mathematics, and technical education in the 20th and 21st centuries* (pp. 133-143). Gadong, Brunei Darussalam: Universiti Brunei Darussalam.

Kanbir, S., Clements, M. A., & Ellerton, N. F. (in press). *Using design research and history to tackle a fundamental problem with school algebra.* New York, NY: Springer.

Owens, K. D., Lean, G. A., Paraide, P., & Muke, C. (2017). *History of number: Evidence from Papua New Guinea and Oceania.* New York, NY: Springer.

Sharp, M. E. P., Farr, G. M., Farr, J. C., & Farr, C. C. (1936). *Early days at St. Peter's College, Adelaide 1854-1878*. Adelaide: The Hassell Press.

University of Melbourne. (1855, March 24). *Registered correspondence, W. E. Hearn to E. G. Mayne*. (Held in the university archives.)

Westbury, I. (1980). Change and stability in the curriculum: An overview of the questions. In H. G. Steiner (Ed.), *Comparative studies of mathematics curricula: Change and stability 1960-1980* (pp. 12-36). Bielefeld, Germany: Institut für Didaktik der Mathematik-Universität Bielefeld.